SUCCESSIVE APPROXIMATION METHOD FOR SOLVING BOUNDARY VALUE PROBLEMS WITH ONE-SIDED NONLINEAR BOUNDARY CONDITIONS

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Annotation. *In this paper, we study the issues of justifying the applicability of the numerical-analytical method of successive approximations [1,2] to the approximate construction of a solution to a boundary value problem for differential equations with nonlinear boundary conditions. One of the most numerous methods for solving boundary value problems is the numerical-analytical method of A.M. Samoilenko [1,2], which has a large number of applications. This article explores the application of this method to solving boundary value problems with one-sided nonlinear boundary conditions.*
Key words: *nonlinear, boundary condition,boundary problem,differential equation,integral equation,sequence of functions,vector function*

МЕТОД ПОСЛЕДОВАТЕЛЬНЫХ ПРИБЛИЖЕНИИ ДЛЯ РЕШЕНИЯ КРАЕВЫХ ЗАДАЧ С ОДНОСТОРОННЕ НЕЛИНЕЙНЫМИ ГРАНИЧНЫМИ УСЛОВИЯМИ

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Аннотация. *В данной работе изучаются вопросы обоснования применимости численно-аналитического метода последовательных приближений [1,2] к приближенному построению решения краевой задачи для дифференциальных уравнений с нелинейными краевыми условиями. Одним из самых многочисленных методов решения краевых задач является численно-аналитический метод А.М. Самойленко [1,2], имеющий большое количество приложений. В этой статье исследуется применение этого метода для решения краевых задач с односторонне нелинейными граничными условиями.*
Ключевое слово. *Нелинейность, граничное условие, краевая задача, дифференциальное уравнение, интегральное уравнение, последовательность функций, векторная функция.*

Let
$$\frac{dx}{dt} = f\left(t, x\right), \tag{1}$$
$$Ax\left(T\right) = \varphi\left(x\left(0\right)\right) \tag{2}$$

Consider the form of nonlinear boundary value problems with an interval to one end. In this problem, before using the numerical-analytical method (1) , the differential equation was its equivalent

$$x\left(t\right) = x_0 + \int_0^t f\left(s, x\left(s\right)\right)ds$$

can be written in the form of an integral equation and using this equation(1),(2) to solve boundary value problems

$$x\left(t\right) = x_0 + \int_0^t f\left(s, x\left(s\right)\right)ds + \alpha t \tag{3}$$

looking for an expression in the form of some α-parameters, here $x_0=\left(x_1(0),x_2(0),...,x_n(0)\right)$, $\alpha=\left(\alpha_1,\alpha_2,...,\alpha_n\right)$. (3) and (2) refer to the boundary value problems α select the parameter:

$$A\left[x_0-\int_0^t f\left(s,x(s)\right)ds+\alpha t\right]=\varphi(x_0),$$

$$\alpha=\frac{1}{T}A^{-1}\varphi(x_0)-\frac{1}{T}x_0-\frac{1}{T}\int_0^t f\left(s,x(s)\right)ds$$

α this definition in (3) put instead

$$x(t)=x_0+\int_0^t f\left(s,x(s)\right)ds-\frac{t}{T}\int_0^T f\left(s,x(s)\right)ds+\frac{t}{T}A^{-1}\varphi(x_0)-\frac{t}{T}x_0$$

or simplify the expression

$$x(t)=x_0+\int_0^t\left[f\left(s,x(s)\right)-\frac{1}{T}\int_0^T f\left(s,x(s)\right)ds\right]+\frac{t}{T}\left[A^{-1}\varphi(x_0)-x_0\right] \quad (4)$$

then we will have such kind of integral equation. This integral equation $x(0)$ given in the definition of parameter (1),(2) will be equal to boundary value problems. There fore, to solve this boundary value problem, (4) we replace the solution of the integral equations.

(4) using the sequence method in the integral equation solving the error we do with the help of this formula

$$x_{m+1}(t,x_0)=x_0+\int_0^t\left[f\left(s,x_m(s,x_0)\right)-\frac{1}{T}\int_0^T f\left(s,x_m(s,x_0)\right)ds\right]ds+\frac{t}{T}\left[A^{-1}\varphi(x_0)-x_0\right]$$

$$.m=0,1,2,3,...,\ x_0(t)=x_0. \quad (5)$$

This is the definition of the sequence $x_m(t,x_0)$ solution of the error in any m and $x(0)$ for parameter (2) satisfies the boundary conditions. There fore our next purpose is (5) consistently defining $x_m(t,x_0)$ function sequence $f(t,x)$ functions defined

$$(t,x)\in[0,T]\times D, \qquad D\subset E_m$$

in area D which do not go out with most numbers and $m\to\infty$ да $\left\{x_m(t,x_0)\right\}$ the function sequence shows the sequencing .

Let $[0,T]\times D$ in area $f(t,x)$ function defined and let it be continuous function then this function this function in this area will be non-edge and in this area $\forall t\in[0,T]$ and x',x'' let the Lipshitz conditions be satisfied, that's

$$\left|f(t,x)\right|\le M$$

$$\left|f(t,x'')-f(t,x')\right|\le K\left|x''-x'\right| \quad (6)$$

inequalities would be appropriate, here M components consists of right numbers n measuring vector, If K is $n\times n$ measuring matrix.

Let its own $\dfrac{MT}{2}+\beta$ with a circle D with a circle will be the most $x_0\in D_f$ multitude should not be empty multitude, that's

$$D_f = D - \frac{MT}{2} - \beta \neq 0$$

Here $\beta = \max_{x_0 \in D_f} \left| A^{-1}\varphi(x_0) - x_0 \right|$

Let $Q = \frac{KT}{\pi}$ the eigenvalues of the matrices must be less than one in absolute approximations, that is, let it be so

$$\left| \lambda_j(Q) \right| < 1, \, j \leq n. \tag{7}$$

if the above (6), (7) conditions are appropriate (1),(2) for solving boundary value error problems (5) can be used formula , error $\{x_m(t, x_0)\}$ solutions in $m \to \infty$ assembled boundary, (5) one can show the integral solution of the equations.

For this, let for primary primacy take $x_0(t, x_0) = x_0$. Here if we from (5) divide (6)

$$\left| x_{m+1}(t, x_0) - x_0 \right| \leq \left| \int_0^t \left[f(s, x_m(s, x_0)) - \frac{1}{T} \int_0^t f(s, x_m(s, x_0)) ds \right] ds \right| + \frac{t}{T} \left[A^{-1}\varphi(x_0) - x_0 \right] \leq$$

$$\leq \left(1 - \frac{t}{T}\right) \int_0^t \left| f(s, x_m(s, x_0)) \right| + \frac{t}{T} \int_0^T \left| f(s, x_m(s, x_0)) \right| ds + \frac{t}{T} \left| A^{-1}\varphi(x_0) - x_0 \right| \leq$$

$$\leq M \left[\left(1 - \frac{t}{T}\right) \int_0^t ds + \frac{t}{T} \int_0^T ds \right] + \frac{t}{T}\beta = M\alpha_1(t) + \frac{t}{T}\beta \leq \frac{MT}{2} + \beta$$

That's why $x_0 \in D_f$, $t \in [0, T]$ for this (5) to define the formula $x_m(t, x_0)$ functions D do not go out of multitude, and this gives us the opportunity to solve the following error. There fore D_f multitude should not be empty multitude if this condition is met x_0 consider that the parameter is in this multitude from (5) formula $\{x_m(t, x_0)\}$ we can get in the required quantity a sequence of solutions of the error.

Now we show the sequence of these solutions of the error. For this $\left| x_{m+j}(t, x_0) - x_m(t, x_0) \right|$ division for $j \geq 1$ evaluating we use the value of Cauchy's self-discipline. For this (6) settling down (5)

$$\left| x_{m+1}(t, x_0) - x_0 \right| \leq K \left[\left(1 - \frac{t}{T}\right) \int_0^t x_m(s, x_0) - x_{m-1}(s, x_0) \right] ds + \frac{t}{T} \int_t^T \left| x_m(s, x_0) - x_{m-1}(s, x_0) \right| ds$$

$$r_{m+1}(t) = \left| x_{m+1}(t, x_0) - x_m(t, x_0) \right|$$

Define the last take this and take it

$$r_{m+1}(t) \leq K \left[\left(1 - \frac{t}{T}\right) \int_0^t r_m(s) ds + \frac{t}{T} \int_t^T r_m(s) ds \right] \tag{9}$$

we will write in this way . From (8) for $m = 0$

$$\left| x_1(t, x_0) - x_0 \right| \leq M\alpha_1(t) + \frac{t}{T}\beta$$

will be from (9)

$$r_1(t) \le M\alpha_1(t) + \frac{t}{T}\beta$$

Will be, the next $r_2(t)$ we will evaluate as below:

$$r_2(t) \le K\left[\left(1-\frac{t}{T}\right)\int_0^t r_1(s)\,ds + \frac{t}{T}\int_t^T r_1(s)\,ds\right] \le$$

$$\le K\left[\left(1-\frac{t}{T}\right)\int_0^t \left(M\alpha_1(t) + \frac{t}{T}\beta\right)ds + \frac{t}{T}\int_t^T \left(M\alpha_1(t) + \frac{t}{T}\beta\right)ds\right] \le$$

$$\le KM\alpha_2(t) + K\beta\alpha_1(t)$$

here

$$\alpha_1(t) = \left(1-\frac{t}{T}\right)\int_0^t ds + \frac{t}{T}\int_t^T ds,\ \ \alpha_2(t) = \left(1-\frac{t}{T}\right)\int_0^t \alpha_1(s)\,ds + \frac{t}{T}\int_t^T \alpha_1(s)\,ds.$$

If we continue the process, then from (9) for $m = 2,3,...$

$$r_3(t) \le K^2 M\alpha_3(t) + K^2\beta\alpha_2(t)$$

$$r_4(t) \le K^3 M\alpha_4(t) + K^3\beta\alpha_3(t)$$

……………………………….

$$r_{m+1}(t) \le K^m M\alpha_{m+1}(t) + K^m\beta\alpha_m(t)$$

will be , here

$$\alpha_{m+1} = \left(1-\frac{t}{T}\right)\int_0^t \alpha_m(s)\,ds + \frac{t}{T}\int_t^T \alpha_m(s)\,ds.$$

If

$$\alpha_{m+1}(t) \le \frac{T^m}{\pi^m}\overline{\alpha}_1(t),\quad \overline{\alpha}_1(t) \le \frac{\pi}{3}\alpha_1(t)$$

Take into consideration, then

$$r_{m+1}(t) \le K^m M\frac{T^m}{\pi^m}\overline{\alpha}_1(t) + K^m\beta\frac{T^{m-1}}{\pi^{m-1}}\overline{\alpha}_1(t) = K^m\frac{T^m}{\pi^m}\left(M + \beta\frac{\pi}{T}\right)\overline{\alpha}_1(t) =$$

$$= Q^m\left(M + \beta\frac{\pi}{T}\right)\overline{\alpha}_1(t).$$

will be like this $\left|x_{m+j} - x_m\right|$ for

$$\left|x_{m+j}(t,x_0) - x_m(t,x_0)\right| \le \sum_{i=0}^{j}\left|x_{m+i}(t,x_0) - x_{m+i-1}(t,x_0)\right| =$$

$$= \sum_{i=1}^{j} r_{m+1}(t) \le \sum_{i=0}^{j-1} Q^{m-i}\left(M + \beta\frac{\pi}{T}\right)\overline{\alpha}_1(t) = Q^m\sum_{i=0}^{j-1} Q^i\left(M + \beta\frac{\pi}{T}\right)\overline{\alpha}_1(t)$$

Appropriate will be the equation. For this

$$\sum_{i=0}^{j-1} Q^i \le \sum_{i=0}^{\infty} Q^i = (E-Q)^{-1}$$

equation and $\lim_{m\to\infty} Q^m = 0$ take into consideration the bounder, then in $m \to \infty$ we take the boundary $\{x_m(t,x_0)\}$ function sequence $(t,x_0)\in[0,T]\times D_f$ if there is the equal measuring assembled sequence can be seen:

$$\lim_{m\to\infty} x_m(t,x_0) = x^*(t,x_0)$$

That, $x_m(t,x_0)$ each of the functions (2) the boundary conditions are satisfied, their boundary $x_m(t,x_0)$ function also satisfies this boundary condition.

Now $x_m(t,x_0)$ with function $x^*(t,x_0)$ and between the boundary function we will estimate the mistake . For this from (10) $j \to \infty$ take the boundary .

$$\left|x_m^*(t,x_0) - x_m(t,x_0)\right| \le Q^m \sum_{i=0}^{\infty} Q^i \left(M + \beta\frac{\pi}{T}\right)\overline{\alpha}_1(t) =$$

$$= Q^m (E-Q)^{-1}\left(M + \beta\frac{\pi}{T}\right)\overline{\alpha}_1(t) \tag{12}$$

And now from (5) $m \to \infty$ take the boundary (11) take into consideration, then $x^*(t,x_0)$ function

$$x(t,x_0) = x_0 + \int_0^t \left[f(s,x(s,x_0)) - \frac{1}{T}\int_0^T f(s,x(s,x_0))ds \right]ds + \frac{t}{T}\left[A^{-1}\varphi(x_0) - x_0\right]$$

you can see the solution for the integral equation. But , the equation (1)

$$x(t) = x_0 + \int_0^t f(s,x(s,x_0))ds$$

be equal to the forces of the integral equation (1), (2) solve boundary value problems, x_0 parameter

$$\Delta x_0 = \frac{1}{T}\left[A^{-1}\varphi(x_0) - x_0\right] - \frac{1}{T}\int_0^T f(s,x(s,x_0))ds \tag{13}$$

requires choosing the transformation of the function vector to zero. That's why $x_m(t,x_0)$ function $x^*(t,x_0)$ the boundary x_0 parameter (13) being zero function vector, that's $\Delta(x_0) = 0$

if we choose the solution of the system of algebraic equations, then $x^*(t,x_0)$ function (1) and (2) will coincide with the exact solution of boundary value problems.

Concluding the taken results, we can give like the theorem below:

Theorem. Let $f(t,x)$ function $[0;T]\times D$ found in this area and being continuous function and that's why (6), (7) conditions should be there. If x_0 first error (13) select the function vector by zero, then (5) $m \to \infty$ boundary, that is to say the function $x^*(t,x_0)$ (1), (2) will be a solution to boundary value problems.

The error between the direct solution and the error solution is estimated by the error equation.

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